Anna SHVETS

Mathematical Bases of the Form Construction in Arvo Pärt's Music

Arvo Pärto kūrinių struktūros matematinis pagrindimas

Abstract
The creative method of the Estonian composer Arvo Pärt is based on a rigorous and strict calculation at all levels of composition. Such a calculative approach of creation is a reflection of the informational society and more specifically – of the logic used in modern programming languages, such as Java, C++, Python, Processing etc. The existing approaches of computational musicology or basically mathematical approaches partly allow to discover and to represent the algorithms under which the compositions were created. However in particular cases it is not sufficient to correctly display these algorithms. Thus, a new method of music representation formalization will be presented. This method is based on the use of statements originating from programming languages to logically represent the occurring processes, including form building, in Pärt's compositions. The described methodological approaches will be applied to the instrumental compositions such as *Cantus in Memory of Benjamin Britten*, *Arbos*, *Tabula Rasa*, *Mein Weg*, *Fratres* and *Spiegel im Spiegel*.
Keywords: Arvo Pärt, computational musicology, algebraic approach, mathematical algorithms of the musical form building, comparative analysis.

Anotacija
Estų kompozitoriaus Arvo Pärto kūrybos technika paremta tiksliais visų kūrinio parametrų apskaičiavimais. Toks kūryboje naudojamas skaičiavimo metodas yra informacinės visuomenės atspindys, konkrečiau tariant, logikos, naudojamos šiuolaikinėse programavimo kalbose, tokiose kaip „Java", C++, „Python", „Processing" ir pan., atspindys. Dabartiniai skaitmeninės muzikologijos metodai arba daugiausia matematiniai metodai leidžia iš dalies atrasti ir pavaizduoti algoritmus, kuriuos naudojant buvo sukurti muzikos kūriniai. Tačiau tam tikrais atvejais nepakanka šių algoritmų vien tik parodyti. Straipsnyje pateikiamas naujas muzikos formalizavimo metodas. Naudojamos programavimo kalbų, kuriomis siekiama logiškai pavaizduoti vykstančius procesus, tarp jų ir Pärto kompozicijų formos konstravimo procesą, formuluotės. Aprašytas tyrimo būdas taikomas analizuojant instrumentines kompozicijas („Cantus Benjamino Britteno atminimui", „Arbos", „Tabula Rasa", „Mein Weg", „Fratres" ir „Spiegel im Spiegel").
Reikšminiai žodžiai: Arvo Pärtas, kompiuterinė muzikos analizė, algebrinis metodas, muzikos formos sudarymo matematiniai algoritmai, komparatyvinė analizė.

Introduction

In previous studies of Arvo Pärt's creative style different approaches confirming the close relation between external graphical ideas and sound implementation within his compositions were used (Shenton, 2012). Such approaches as style analysis, musical hermeneutics, Schenkerian analysis, set theory, triadic transformation and others (Robinson, 2012) could make sense only partially and do not give the key for the entire understanding of Pärt's creative process. The idea of the musical archetypes proposed by Brauneiss (Brauneiss, 2012) fails when the same rhythmic structure organization could be found not only in *Arbos*, for which as the author assumes the structure was designed, but in *Mein Weg*, *Cantus in Memory of Benjamin Britten* and in a mirrored version in *Silentium* from *Tabula Rasa*. Thus, the traditional musicological approaches are not sufficiently adequate for Pärt's compositions analysis.

Instead, the computational musicology approaches may be used. Computational musicology appeared with the appearance of technology in the 20th century and consists of search for a computational aspect of music and development of theoretical models of the interactions between the levels of representation, which use this computational aspect (Ahn, 2009).

One of the approaches of computational musicology is the algebraic approach to music theory which proposes mathematical methods for music analysis (Andreatta, 2003), which have been used for present analysis.

Aim of the article

Despite the aesthetic position of Pärt, assuming that everything that could be mathematized has nothing to do with music (Pärt, 1990), within his own works the composer uses mathematics and any sort of calculation at all levels of music composition – in general form structure, in formation of melodic patterns, in polyphonic relation between voices. Thus the aim of the present article mainly consists of showing these mathematical regularities, especially of the form construction,

on several examples of Pärt's music. An additional task lies in the development of a new methodological approach, allowing representation of music processes as logical statements with the use of semantics coming from modern computer languages, such as Java, C++, Python or Processing.

For the analysis six instrumental compositions were chosen – *Cantus in Memory of Benjamin Britten* (1977), *Arbos* (1977), *Tabula Rasa* (1977), *Fratres* (1977), *Spiegel im Spiegel* (1978) and *Mein Weg* (1989). The main reason for the instrumental compositions selection is their freedom from text and human voices limits, which can add some sort of restrictions. The selection of the specified compositions is made under their constructional comparability, essential for comparative analysis.

Mapping principle

Computational analysis requires the mapping of notes to numbers. This mapping consists of the numerical values assignment to the each step of the scale, also sign "+" or "-" assignment depending on where these steps are situated – above the starting note or below. The note considered as the starting point of voice movement receives the value of number "0", the note of one step above receives the value of number "1" with sign "+" and the note of one step below – the value of number "1" with sign "-", similarly the note of two steps above – the value of number "2" with "+" sign and the note of two steps below – value of number "2" with minus sign etc.

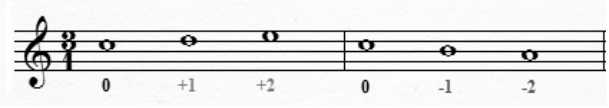


Fig. 1. Mapping method illustration

Such mapping method was used to Pärt's compositions in previous works for analysis of *Mein Weg* structure (Shvets, 2012), computer modelling of *Spiegel in Spiegel* (De Paiva Santana, 2012). The attempts of this method's application have been used to the *Fratres* structure analysis (Zamornikova, 2011), but inappropriate values assignment caused the absence of significant result of the method application: all the steps above the "0" note were noted as number "1" and all the steps below – as number "1" with "-" sign. Thus, the ranges affecting the structure configuration were not received.

Form building algorithms

In numeric representation

The construction of the form in Pärt's compositions is far from traditional musical forms organization and relies on the main melodic voice development. Sometimes the composer uses the measure structures grid, guided, in turn, by mathematical logic. Within this grid the composer inserts the main music material (*Fratres*), which can be alternated with secondary-level music material (*Ludus*). All these cases will be described in detail.

The main Melodic voice's (M-voice) development consists of linear algebra operations, such as addition, subtraction and multiplication. The symmetry laws also play a significant role in the M-voice construction and can be applied in vertical (change of sign for the same range) and horizontal senses (reversed order of numbers). These algorithms are visible on the micro level of each piece construction and on the macro level between pieces, where the structure of the one piece could appear as a reversed or altered version of another piece.

The micro level of each piece composition is constructed from larger and smaller structures, subordinated hierarchically between them. These larger and smaller structures are classified as stanzas, phrases and elements. In some cases intermediate structures as semi-stanzas or semi-phrases can appear, but it is not obligatory and depends on each case.

Stanza is a completed event within Part's music. It contains the beginning and the end of the structural numerical range on which the mathematical operations are executed. Stanza can or cannot be divided into smaller structures – the quantity of used numerical operations and transformations decide for it.

The use of mathematical operations within each structure has several levels of interactions:

- Stanza-stanza level;
- Phrase-phrase level;
- Element-element level.

The implementation of logic operations will be considered now in concrete examples. For this implementation only a few first stanzas of the main M-voice of each piece will be shown to set the principle of the form algorithm formation. The whole ranges in graphical representation will be shown later.

The interactions on stanza-stanza level are inherent in the structure of *Fratres*. In this case the replacement of elements within the phrases of neighbouring stanzas occurs (-1 +1 towards +1 -1).

Fratres		
	1st stanza	2nd stanza
1st p.	0 - 1 + 1 0	0 + 1 - 1 0
2nd p.	0 - 1 - 2 + 2 + 1 0	0 + 2 + 1 - 1 - 2 0
3rd p.	0 - 1 - 2 - 3 + 3 + 2 + 1 0	0 + 3 + 2 + 1 - 1 - 2 - 3 0

The interactions on the phrase-phrase level are inherent in the *Mein Weg* and *Spiegel im Spiegel* structure. These cases are rich of mathematical transformations such as:

Vertically mirrored ranges within the phrases (+2+1 towards -2 -1) of the same stanza;

Horizontally mirrored ranges of numbers with the use of linear addition: +1 +2 +3 towards (+4) +3 +2 +1 (or the same range of numbers with "-" sign).

	Mein Weg		
init		0 +1	-1 0
1st st.	1st p.	+2 +1 0	-2 -1 0
	2nd p.	+1 +2 +3 0	-1 -2 -3 0
2nd st.	1st p.	+4 +3 +2 +1 0	-4 -3 -2 -1 0
	2nd p.	+1 +2 +3 +4 +5 0	-1 -2 -3 -4 -5 0

	Spiegel im Spiegel		
1st st.	1st p.	-1 0	+1 0
	2nd p.	-2 -1 0	+2 +1 *0*
2nd st.	1st p.	-1 -2 -3 0	+1 +2 +3 *0*
	2nd p.	-4 -3 -2 -1 0	+4 +3 +2 +1 0

The interactions on the element-element level are inherent in both parts of *Tabula Rasa*, in *Arbos* and *Cantus in Memory of Benjamin Britten*. These interactions contain the following mathematical transformations:

Mirrored order or numbers within the elements of phrases (+1+2+1) and vertically mirrored phrases (with change of sign);

Replacement of the numbers' order with linear addition of the elements to each new stanza.

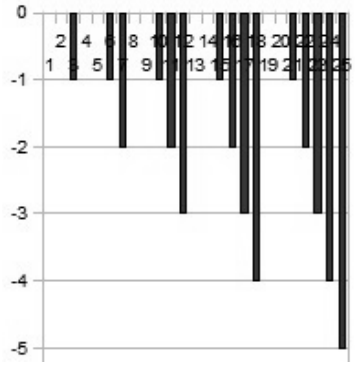
Ludus		
	1st phrase	
	1st element	2nd element
1st stanza:	0 +1	0 -1 0
2nd stanza:	0 +1 +2 +1	0 -1 -2 -1 0
3rd stanza:	0 +1 +2 +3 +2 +1	0 -1 -2 -3 -2 -1 0
	2nd phrase	
	1st element	2nd element
1st stanza:	0 -1	0 +1 0
2nd stanza:	0 -1 -2 -1	0 +1 +2 +1 0
3rd stanza:	0 -1 -2 -3 -2 -1	0 +1 +2 +3 +2 +1 0

Silentium		
	1st phrase	2nd phrase
1st stanza	0 +1	0 -1
2nd stanza	0 +1 +2 +1	0 -1 -2 -1
3rd stanza	0 +1 +2 +3 +2 +1	0 -1 -2 -3 -2 -1

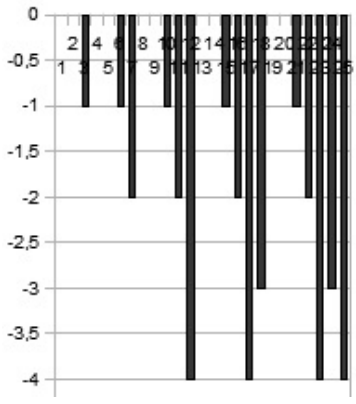
Arbos		***Cantus***
1st stanza	0 -1	0 -1
2nd stanza	0 -1 -2	0 -1 -2
3rd stanza	0 -1 -2 -4	0 -1 -2 -3
4th stanza	0 -1 -2 -4 -3	0 -1 -2 -3 -4
5th stanza	0 -1 -2 -4 -3 -4	0 -1 -2 -3 -4 -5

In graphical representation

All the ranges of numbers presented above can be represented also with graphics as histograms. The graphical representation will clear up the similarities of the structure algorithms between the pieces. With graphical representation it becomes obvious that the structure algorithm of *Arbos* is a slightly altered version of *Cantus in Memory of Benjamin Britten*:

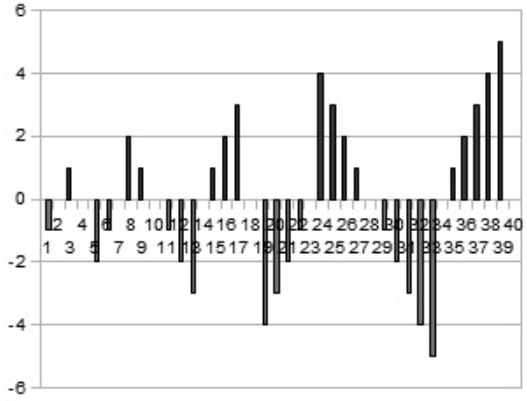


Cantus in Memory of Benjamin Britten

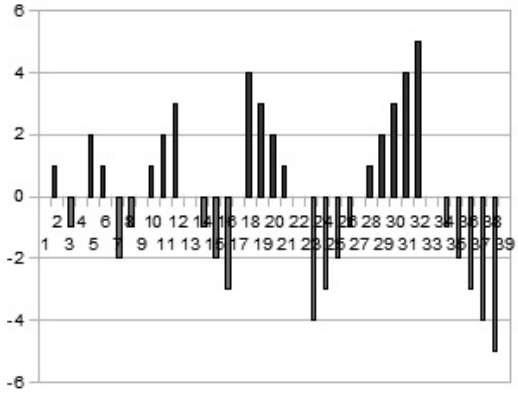


Arbos

It is also obvious that the form structure algorithm of *Mein Weg* is a reversed version of *Spiegel im Spiegel*:

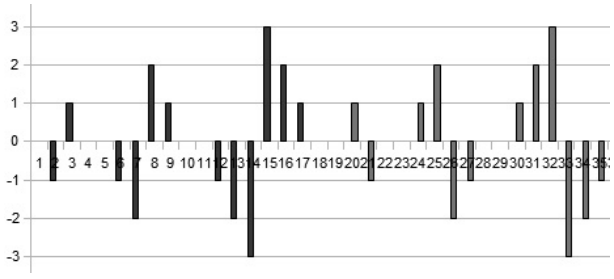


Spiegel im Spiegel

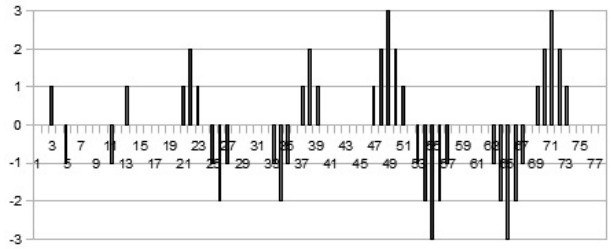


Mein Weg

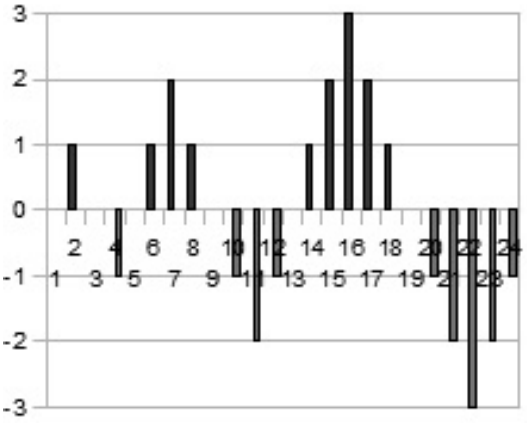
The idea of question-answer relations appears in the *Ludus* part of *Tabula Rasa* (element-element level) and *Fratres* (phrase-phrase level); *Silentium* part of *Tabula Rasa* appears as a simplified version of *Ludus*.



Fratres



Ludus



Silentium

In formulas representation

The application of algebra operations with symmetry laws allows us to represent the M-voice development as mathematical formulas or conditions of a computer program. The code presentation for the formalization of music representation is a new methodological approach that can be used as alternative to verbal or graphical representations, but in conditions of information society appears as the most compact and appropriate (Lyotard, 1979). It is a logical level of music representation. For example, the form of *Cantus in Memory of Benjamin Britten* can be represented as the *for loop* statement, the main logic of which allows executing repeated iterations of values. The *for loop* statement in programming is usually used when the amount of iterations is predefined before entering the loop:

for ($i = 0$; $i < x$; i++) {
$x = x * (-1)$}

This simple code will add negative numbers in ascending order from 0 ($i = 0$) to x ($i < x$) until the value of x is reached (i++ means proceeding with a natural range numbers in ascending order – 1,2,3,4,5 etc.). The "for" operator means that we want to make the iteration of number i, which appears in condition of the statement. The assignment of x to x multiplied by (-1) means that each item of the range that we'll receive will be with "-" operator. This code returns numerical range of the form development algorithm shown in numeric representation for *Cantus in Memory of Benjamin Britten*.

The *for loop* statements are useful for representation of the final numeric values of the range's development. The most important x-value within the loop (the final point of ranges increase) can be also presented in terms of mathematics as n-point of range increase.

Earlier the differences of the form development algorithms on first few stanzas example were shown. However the whole form representation requires the final value to be considered in its entirety. If we consider that we've been analyzing only the main M-voice development of one layer and that each piece contains at least more than one layer (but not more than 5 in analyzed works), it becomes obvious that traditional descriptive methods are not efficient in the processing of such amount of information. Only the codes' representation with the initial and final set of values could resolve the problem of large quantity of information processing to be used in comparative analysis. The initial set of values for the main M-voice is especially useful in comparison to different M-voices within the same piece and will be shown later.

The graphical visualization of the whole main M-voice structure allows us to follow farther form algorithms' development until the formation of the entire structure. Figures 2, 3, 4, 5, 6, 7 and 8 present graphical visualizations of the whole main M-voice from selected Pärt's compositions. The comparison of these graphical representations clarifies that similar initial algorithm of development can result in

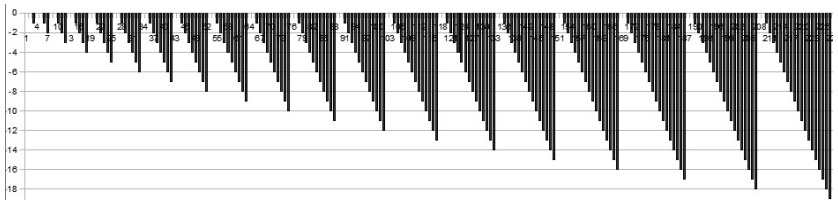


Fig. 2. The whole range of the main M-voice from *Cantus in Memory of Benjamin Britten*

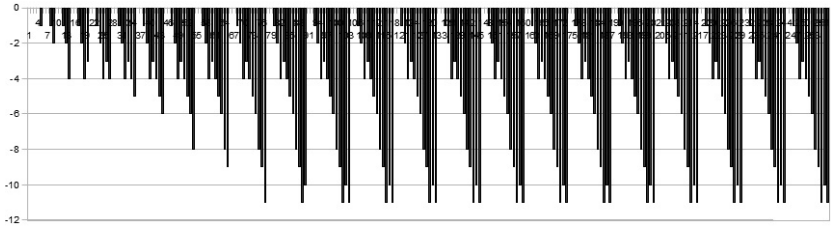


Fig. 3. The whole range of the main M-voice from *Arbos*

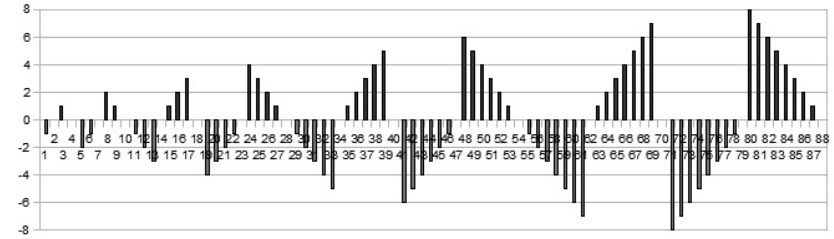


Fig. 4. The whole range of the main M-voice from *Spiegel im Spiegel*

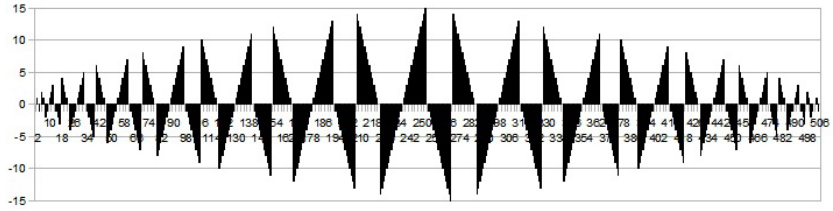


Fig. 5. The whole range of the main M-voice from *Mein Weg*

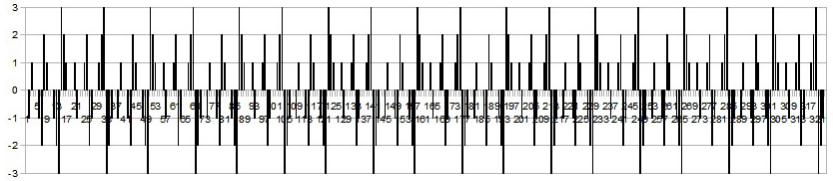


Fig. 6. The whole range of the main M-voice from *Fratres*

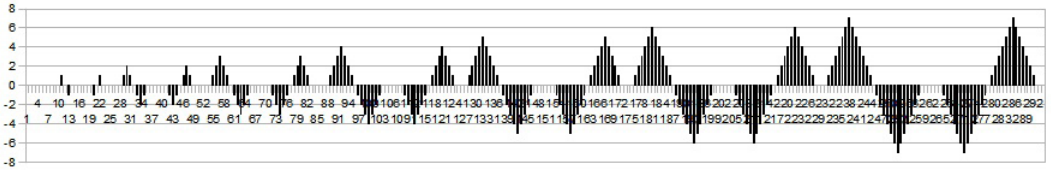


Fig. 7. The whole range of the main M-voice from *Ludus*

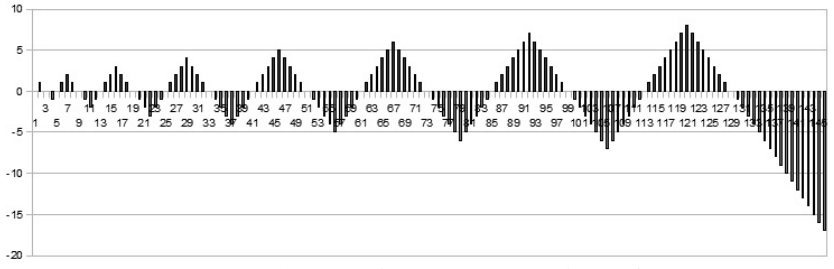


Fig. 8. The whole range of the main M-voice from *Silentium*

different forms configuration according to the following parameters:

1) Number of stanzas;

2) Number of repetitions of the same structure.

Let us present now the ways of development after reaching the highest *n*-point of range increase in graphical representation. The common feature for all seven main M-voice forms is aspiration to achieve the *n*-point of number range, which also is the final point in the form development. The form of the main M-voice from *Mein Weg* is an exception, where after the increase until the highest *n*-point of ranges, gradual decrease back to *0* occurs.

The exact repetitions of stanzas in case of *Fratres* or *Arbos* don't give such clear visual representation of this main feature, however previous numeric representations have shown the algorithms with the use of linear addition until the *n*-point of ranges is reached. To formalize the whole structure of these pieces part of a code can be used. The occurred repetitions after the primer algorithm execution can be presented as *if statement*. It means that we will follow the primer algorithm until it reaches *n*-point of range and if it reaches that point, we will repeat the last stanza (in case of *Arbos*) or the first and the second stanza (in case of *Fratres*) *x* times:

if (*n*-point = stanza[*i*])
stanza[] * *x*

The result of the number of stanzas change is visible on the example of the comparison of the main M-voice structures from *Spiegel im Spiegel* and *Mein Weg*: the *n*-point of the main M-voice from *Spiegel im Spiegel* equals to +3/-3 and the *n*-point from *Mein Weg* equals to +15/-15.

This parameter is important for comparison of the other M-voices with the main M-voice within the same piece. The set of these data allows us to take into consideration the entire form of the piece and to consider the relations between M-voices in different layers. These data can be visualized further.

Types of complementarity

The ways of vertical complementarity in analyzed pieces is reached by means of three methods:

1) two times rhythmic values augmentation of each next M-voice within the same piece (*Cantus in Memory of Benjamin Britten, Arbos, Mein Weg, Silentium*);

2) Dispersion of others M-voices in accompaniment figures (*Spiegel im Spiegel*, partly *Ludus*);

3) Canonic shifting of the M-voices containing the same rhythmic values (*Ludus*).

The vertical complementarity is not a single type of complementarity which can be found in Arvo Pärt's music. The horizontal complementarity also occurs with measure structures division mentioned above. There are two cases of such measure structures in selected compositions, which differ by its use.

The first level of use consists of mathematical operations that could be performed with these structures, but it does not affect the general structure of M-voices. This level of measure structures division is used in the *Fratres* and *Ludus* part from *Tabula Rasa*.

The second level consists of influencing M-voices development because measure structures play the role of frame slots, which can be filled out only with a special type of information. Each measure structure has its own set of M-voices and they are developing independently within separate measure structures. The second level of measure structures division is inherent in *Ludus* part from *Tabula Rasa*.[1]

Mathematical operations performed on measure structures in *Fratres* consisting of addition of the number "2" in order to receive metric quantity units ("9" as result of 7+2 and "11" a 9+2 additions) were described earlier (Zamornikova, 2011). The number "2" appears also in multiplication operations on micro (repetition of one meter "6/4") and macro levels (repetition of a number, or a *list* of meters "7/4 9/4 11/4").

The laws of internal and external symmetry are also inherent in the measure structures construction. The internal symmetry consists of the number "9", which is situated as a metric quantity unit in the middle of the *list* of metric units and farther is used as the number of repetitions for stanzas. The external symmetry consists of the use of meter "6/4" at the beginning and the end of the piece. The whole measure structure can be presented as a mathematical formula with described operations:

{ [6/4*2 (7/4 9/4 11/4)*2]*9 6/4*2 }

Before describing mathematical operations on measure structures used in *Ludus* part from *Tabula Rasa*, let us define its general measure structures construction. *Ludus* can be devised into two main parts – before cadenza and after. The part before cadenza in a dramaturgical sense is a gradual increase of tension before cadenza-culmination. Each of these greater parts have internal division: part before culmination has four stanzas (devised into 8 semi-stanzas) and cadenza-culmination part consists of two internal

[1] Some researches (Zamornikova, 2011) assume the presence of such level use in *Fratres*. They believe that such division occurs between the S-part (M-voice stanzas) and R-part, consisting of the two measures filled out with *ostinato* percussion rhythmic figure. But as we will see later in comparison with other works by the same composer, the percussion insertion usually plays the role of "points" or "separators" at the end of stanzas and is not treated as independent material, pretending to be divided from the main M-voice.

culminations – first with fast decay in a-moll ambience and second with fast dynamic increase in ambience of the diminished seventh chord (VII_7 to g-moll, the subdominant function to d-moll, tonality of the next part *Silentium*). The mathematical operations on measure structures are used in the part before culmination.

Mathematical operations in *Ludus* consist of addition and subtraction. Addition is used for adding measures and subtraction for subtracting meter units. Each stanza consists of two semi-stanzas related by question-answer relations. Each semi-stanza contains three measure structures. Question-answer relations are inherent in only the third measure structure (MS3). The measure structures division is made under meter change: each first semi-stanza of each stanza begins with 6/4 meter and each second semi-stanza of each stanza – with 5/4 followed by 6/4 meter. Measures filled with these meters make part of the MS1. The meter of the second measure structure (MS2) is constantly changing being affected by the subtraction operator. The third and the last measure structure (MS3) consist of measures with 4/4 meter. The measure structure principle of division described is shown in the following table:

Ludus measure structures division									
		MS1		*Comment for MS1*	**MS2**	*Comment for MS2*	**MS3**	*Comment for MS3*	
1st stanza	1st s.-stanza	6/4		*0*	8/2 G.P.		4/4 (6)	*+6*	
	2nd s.-stanza	5/4	6/4	*1*	7/2 G.P.	*-1m. unit*	4/4 (10)	*+4*	*+6*
2nd stanza	1st s.-stanza	6/4 (3)		*+3 measures*	6/2 G.P.	*-1m. unit*	4/4 (12)	*+2*	
	2nd s.-stanza	5/4	6/4 (4)	*+3 measures*	5/2 G.P.	*-1m. unit*	4/4 (16)	*+4*	*+6*
3rd stanza	1st s.-stanza	6/4 (6)		*+3 measures*	4/2 G.P.	*-1m. unit*	4/4 (18)	*+2*	
	2nd s.-stanza	5/4	6/4 (7)	*+3 measures*	3/2 G.P.	*-1m. unit*	4/4 (22)	*+4*	*+6*
4th stanza	1st s.-stanza	6/4 (9)		*+3 measures*	2/2 G.P.	*-1m. unit*	4/4 (24)	*+2*	
	2nd s.-stanza	5/4	6/4 (10)	*+3 measures*	1/2 G.P.		4/4 (30)	*+6*	

Comment sections show the regularities, which can be formalized as pieces of codes.

Addition of measures in MS1 can be displayed as a *while loop* statement (the logic of *while loop* statement consists of code execution until the condition is true) with a different initial value for the first and the second semi-stanzas within each stanza: for the first semi-stanzas initial value of x before addition is "0" and for the second semi-stanzas the initial value of x equals to "1":

a) While loop for the first semi-stanzas:

```
x = 0
while ( x  <= 9){
x+3}
```

b) While loop for the second semi-stanzas:

```
x = 1
while ( x  <= 10){
x+3}
```

The first *while loop* will return values *3*, *6*, *9*, which are the numbers of added measures for the first semi-stanzas of the second, third and fourth stanzas consequently. The second *while loop* will return values *4*, *7*, *10*, which are the numbers of added measures for the second semi-stanzas of the same stanzas as in case of the first semi-stanza's addition.

Subtraction operation used in MS2 section can be expressed with the already described *for loop* statement:

```
for (i = 8; i >= 1; i-- ) {...}
```

This code performs gradual decrement (by *-1*) from value *8* to *1* and it reflects a gradual decrement of quantitative metric units in MS2 in each semi-stanza. Thus it begins with meter 8/2 in the first semi-stanza of the first stanza and it finishes with meter 1/8 in the second semi-stanza of the fourth stanza, which is the last semi-stanza of the form structure before cadenza.

Addition regularities in the MS3 are guided by symmetrical laws: the first semi-stanza contains six measures, the last semi-stanza has number six as the quantity of added measures (comparing to the previous seventh semi-stanza). The addition between these first and eighth semi-stanzas can be expressed with *(4+2) * 3*, because the quantity of measures added to the second semi-stanza equals to *4* and to the third – to *2*. The same *4* followed by *2* added measures are repeated yet in the pairs of 4th–5th and 6th–7th semi-stanzas. The whole formula for addition regularities in the MS3 has the following expression:

```
{6 [(4+2) * 3] 6}
```

It is easy to perceive that *4+2* also equals to *6*, thus the divided internal number "6" is surrounded by external whole number "6", creating perfect symmetry.

Symmetry laws are inherent in the canonic shifting of the M-voices (with aligned T-voices) containing the equal rhythmic values in the same MS3 of *Ludus*: descending order is followed by mirrored symmetrically equal ascending

order in each semi-stanza. The question-answer relations between first and second semi-stanzas occurs on this level too: the first semi-stanza contains two phrases with shifted pairs of M and T voices, while the second semi-stanza contains only two pairs of M and T voices, but instead additional M-voice within each of its two phrases. The described regularities can be expressed by means of formula where the dash signs ("-") means simultaneity and plus signs ("+") complementarity or shifting:

1st phrase *2nd phrase*

1st s.-st.: (M-T)*2 + (M-T) | (T-M) + (M-T)*2

2nd s.-st.: (M-T)*2 + (M+M) | (M+M) + (M-T)*2

Going deeper into phrase-phrase relations within each semi-stanza of MS3 from the *Ludus* part of *Tabula Rasa* other addition regularity appears. This time it affects the quantity of added quarters in bound between phrases. The bound consist of repeated quarters on the same pitch "A2" with *staccato* articulation signs. The quantity of quarters in the first semi-stanza equals to *10* and in the second semi-stanza – to *12*, thus the quantity of added quarters between two semi-stanzas within the same stanza equals to *2*. This regularity will be held for the rest of other pairs of semi-stanza. The number of added quarters between each of stanzas is equal to *6*, thus if the quantity of bound measures in the second semi-stanza within the first stanza was *12*, the quantity of these measures in the first semi-stanza within the second stanza is *18*. It results in 4 times 2 quarters added in pair of semi-stanzas and 3 times 6 quarters added between four stanzas. This regularity can be expressed with *nested for loop,* where outer iteration (+6 or *i*) is executed after inner iteration (+2 or *j*). The general structure of *nested for loop* is as follows:

for (*i* = 0; *i* < num1; *i*++) {
for (*j* = 0; j < num2; *j*++) {...}}

In our case each outer *i* (+6) addition will be done after the inner *j* (+2) addition is executed.

Rhythm of M-voices

Form organization depends on rhythmic organization in both vertical and horizontal senses. Vertical rhythmic organization mentioned earlier as the first method of vertical complementarity relies on 2 times rhythmic values augmentation of each next M-voice within selected structure. The horizontal organization also uses the principle of 2 times augmented/reduced rhythmic values if they are not homogeneous. For visualization we will apply such values signification: 0.5 = eights, 1 = quarter, 2 = half note, 4 = whole note, 8 = double whole note, 16 = quadruple whole note, 32 = octuple whole note; 1.5 = dotted quarter, 3= dotted half note, 6 = dotted whole note.

Mein Weg

Cantus in Memory of Benjamin Britten

Arbos

Silentium

Fratres

Spiegel im Spiegel

The described principle of two times rhythmic values augmentation is used for *Mein Weg*, *Cantus in Memory of Benjamin Britten* and *Arbos*. The vertical rhythmic construction of *Silentium* appears as reversed copy of *Arbos*. M-voices inherent in *Fratres* and *Spiegel im Spiegel* are guided by equivalency principle in their rhythmic cells. The formula *1*n* in *Fratres* means that the quantity of quarters varies according to the used meter (7/4, 9/4, 11/4), but first half note (*2*) and last dotted half note (*3*) remains in each metric cell, creating the structure of frame with core. This representation does not negate the second type of vertical complementarity used in *Spiegel im Spiegel*, only it shows that two dispersed in accompaniment M-voices have the same harmonic rhythm duration, despite their non-synchronic entry.

The *Ludus* part is not shown within these representations because of its horizontal complementarity of the second type (which affects the general form structure construction) instead of vertical and will be analyzed in other way. It is reasonable to analyze each measure structure separately by proposing the whole tables of rhythmic development, because it differs from the first or second semi-stanza type.

First measure structure (MS1) contains only one M-voice which consists of the same numeric ranges that M-voices from MS3. However the rhythmic organization differs a lot – from rhythmic values to rhythmic progressions. Using the same numeric representation of rhythmic values, let us present the rhythm structure for MS1 with the following table:

Ludus MS1		
1st stanza	1st s.-st.	6
	2nd s.-st.	2 + (3 * 3)
2nd stanza	1st s.-st.	(1.5 * 4) + (3 * 4)
	2nd s.-st.	2 + (3 * 3) + (1.5 * 4) + (3 *4)
3rd stanza	1st s.-st.	((1.5 * 4) + (3 * 4)) * **2**
	2nd s.-st.	2 + (3 * 3) + ((1.5 * 4) + (3 * 4)) * **2**
4th stanza	1st s.-st.	((1.5*4)+(3*4))***3**
	2nd s.-st.	2 + (3 * 3) + ((1.5 * 4) + (3 * 4)) * **3**

The multiplication sign means repetition of the same rhythmic values and plus sign – simple neighbouring of different rhythmic values. If we store the initial rhythmic value *6* (dotted whole note) from the first semi-stanza of the 1st stanza in x_0 and its following rhythmic series corresponding to the formula *(1.5 * 4) + (3 * 4)* in *x*, then if we store the range of rhythmic values from the second semi-stanza of the 1st stanza and corresponding to the formula *2 + (3 * 3)* in *y*, we receive a much more simplified version of the same table:

Ludus MS1		
1st stanza	1st semi-stanza	x_0
	2nd semi-stanza	y
2nd stanza	1st semi-stanza	x
	2nd semi-stanza	y + x
3rd stanza	1st semi-stanza	x * 2
	2nd semi-stanza	y + (x *2)
4th stanza	1st semi-stanza	x * 3
	2nd semi-stanza	y + (x * 3)

The rhythmic range corresponding to *x* is present in every semi-stanza from the second stanza. The range stored in *y* plays the role of the beginning of entire rhythmic range inherent in the second semi-stanza. The formula of rhythmic progression for whole stanza looks as follows:

(x * n) + (y + (x * n))
where *n* is a number of repetitions from 1 to 3.

The rhythmic organization of MS2 directly depends on metric units subtraction presented earlier and consists only of pauses (Grand Pauses), thus the rhythmic values substruction is exactly the same as in case of metric units (one metric unit equals to one half note).

The MS3 section contains multiple M-voices which can be divided into two groups regarding used rhythmic progressions – static and dynamic. The static group contains rhythmically identical M-voices consisting of quarters (*1*). The quantity of M-voices within the static group changes from three to four upon first to second semi-stanza's type use. The dynamic group contains three M-voices which rhythmically vary between them and inside themselves. The first M-voice generally consists of eighths (*0.5*) with the possible insertion of two sixteens on feeble times. The second M-voice consists of triples (*0.3*) and the third M-voice – of sixteens (*0.25*) only[2].

A general table of rhythmic values within the dynamic group of MS3 from the first semi-stanza looks as follows:

Ludus MS3 dynamic group	
1st M-voice:	(0.5 * 2) * 3 + dot
2nd M-voice:	(0.3 * 3) * 3 + dot
3rd M-voice:	(0.25 * 4) * 3 + dot

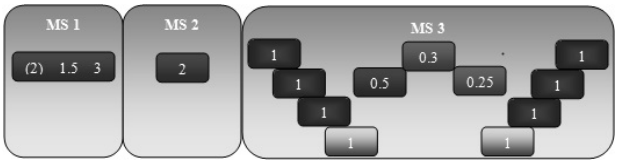
The formulas in parenthesis correspond to one quarter with an appropriate number of repetition of smaller values in it. The multiplication of values in parenthesis with a specified number means the repetition within the same measure where "dot" is included. Each next semi-stanza adds one more measure to each M-voice of this group before primarily appeared progression. Thus, the development of all rhythmic progressions for all M-voices of dynamic group can be formalized as follows:

((((value*n)*4)*n) + ((value*n)*3+dot)) *n

Or if represent the formula for added measures ((values*n)*4) as *A* and finishing formula ((value*n)*3+dot) as *F*, we receive the next formula:

(A * n + F) * n

Considering the dynamic changes in rhythmic progressions from three analyzed measure structures of *Ludus*, the visual representation of its horizontal rhythmic structure will be done with use of values in discovered formulas and on example of one semi-stanza after the regularities are set already (second stanza):



Ludus

[2] The bigger values that can appear at the end of each semi-stanza for each M-voice are considered as "dots". Thus, their rhythmic values are not taken into consideration.

Dynamic changes in rhythmic progressions based on mathematical laws are inherent also in the horizontal rhythmic structure development of *Arbos*. The regularity of two times augmentation/reduction was described already, but within this piece the combinatorics plays decisive role in domain of rhythmic development. At first sight the combinatorics finishes on a simple regular replacement of bigger rhythmic value by smaller rhythmic value, creating alternation of iambic and choreal rhythmic structures in each new stanza. However the first and the last rhythmic values of each stanza create all the possible combinations of two elements – *a* and *b*, when the combinatory possibilities are exhausted (after four stanzas), a new cycle with the same order of combinations begins:

Arbos rhythmic structure combinatorics	
st stanza	**ba**
2nd stanza	**aa**
3rd stanza	**ab**
4th stanza	**bb**

Described rhythmic combinatorics is inherent in all M-voices of the piece. The values of *a* and *b* in the three M-voices differ from the applied law of vertical two times augmentation/reduction regularity.

Similar mathematical regulations are used in development of the rhythmic structure of *Mein Weg*. In this case we will count the number of repetitions of uniform rhythmic values for each new number from pitch range, present in each M-voice. After one measure of primer initialization of movement, the regularities are set. Each stanza contains three rhythmic sections. These rhythmic sections are separated by a pause equal to one rhythmic value inherent in specified M-voice (eighth pause for the highest M-voice, quarter pause for the middle M-voice and half note pause for the lowest M-voice):

Mein Weg rhythmic structure development			
	1st section	2nd section	3rd section
init.	(2 4)*2 2		
1st st.	3 (2 4)	1 (4 2)*2 4	1 (4 2) 4
2nd st.	3 (2 4)*2	1 (4 2)*4 4	1 (4 2)*2 4
3rd st.	3 (2 4)*3	1 (4 2)*6 4	1 (4 2)*3 4

Repetition structure in the second and third sections consists of the frame (1 4) and core (4 2). The number of repetition of the core in the second section equals to the formula: *x**2, in comparison to the third section. The structure of the frame with varied core is familiar to *Fratres* where it was inherent in the horizontal proportions of rhythmic values. Let us formalize the numbers received using following assignments:

- for rhythmic values repetitions: 1 = a, 2 = b, 3 = c; 4 = d;
- for core repetitions (after multiplication sign): N = natural range of numbers from 1 to 7 and in back order (increment up to the n-point of ranges occurs until 7th stanza with gradual decrement until 14th stanza); E = even range of numbers, which are two times taken a natural range of numbers from 1 to 7;

After assignment the next formula for homogeneous rhythmic values within each M-voice of *Mein Weg* appears:

c (b d)*N | a (d b)*E d | a (d b)*N d

Clear symmetrical succession of core multipliers can be extracted from the previous formula:

N – E – N

After reduction of repeated elements, the next formula appears:

c (b d)*N | a (d b)*E/N d |

where the slash sign "/" means "or after". The reversed succession of elements within the core in the first section compared to next two sections to the use of reversion method for numeric range of pitches iterations widely used in general M-voice construction in *Mein Weg*.

The number of repetition creates rhythmic groups to which the repeated rhythmic values belong. It means that one time (plus 8th pause) or two times repeated eighth belong to quarter (*1*) and three times repeated (plus 8th pause) or four times repeated eighths belong to a bigger rhythmic value – half note (*2*). The formula of rhythmic groups for repeated homogeneous rhythmic values looks as follows:

2 (1 2)*N | 1 (2 1)*E 2 | 1 (2 1)*N 2

Representing the core part *(1 2)*N* with their repetitions as *a* and the *(2 1)*E/N as b*, we receive the following formula view:

2 a / 1 b 2 / 1 b 2

Clear alternation of bigger value with smaller appears:

2 1 2 1 2

Rhythmic sections correspond to the numeric range movements: movement from a greater even number with the plus sign to the smallest odd number (+2 +1) correspond to the first section, movement from the smallest odd number with the minus sign to a greater odd number (-1 -2 -3) corresponds to the third section, movements from a greater even number with the minus sign to the smallest odd number (-2 -1) and from the smallest odd number with the plus sign to a greater odd number (+1 +2 +3) are

split together in the third section. The formalization of the regularity described looks as follows:

1st section	$+E_g => +O_s$
2nd section	$-E_g => -O_s \mid +O_s => +O_g$
3rd section	$-O_s => -O_g$

Certain combinatory regularities are visible in this formalization too, if we consider a few restrictions:

- The numbers of one numeric range movement have the same sign as the first number of the range;
- The change of sign between numeric range movement are guided by strict alternation;
- Even greater number always moves to the smallest odd number and the smallest odd number to a greater odd number.

An explanation for two movements split in one (second) rhythmic section is evident with mathematical union – the second section contains the mathematical union of the first and last numeric range movements. Storing $+/-E_g$ in *A*, $+/-O_s$ in *B* and $+/-O_g$ in *C* we receive the following formalization view:

1st section	A B
2nd section	A B U B C
3rd section	B C

T-voice

Tintinnabuli-voices (T-voices) in the analyzed pieces are dependent on its vertical and horizontal correlations with M-voices.

Vertical correlations

1. Formula correlation:
- The same for each layer (*Cantus in memory of Benjamin Britten*);
- Differs from layers, but is held within each layer (*Mein Weg*);
- Differs from layers and within each layer (*Fratres, Arbos, Silentium, Ludus, Spiegel im Spiegel*);

2. Rhythmic correlation:
- Rhythmic values of M-voices are duplicated in corresponding T-voices (*Cantus in Memory of Benjamin Britten, Arbos, Mein Weg, Fratres, Ludus*);
- Rhythmic values of M-voices are not duplicated in corresponding T-voices (*Silentium, Spiegel im Spiegel*);

Let us present the graphically described cases combining formula (in white) and rhythmic correlations (in yellow):

Mein Weg

Cantus in Memory of Benjamin Britten

Arbos

Silentium

Fratres

Spiegel im Spiegel

Ludus

The quantity of T-voices differs from the piece, and usually equals to one T-voice per one M-voice, but two pieces make an exception – *Arbos* with two T-voices per one middle M-voice and *Fratres* with one T-voice for two M-voices.

The pieces regarding formulas of T-voice additions can be summarized to the following cases:

Entirely based on the formula T-/+1 for the T-voices addition (*Ludus, Cantus in Memory of Benjamin Britten*);

Based on the formula T-/+1 for the T-voices addition with T+2 incrustations (*Mein Weg, Spiegel im Spiegel*);

Entirely based on the formula T-2 (in relation to the highest M-voice) for the T-voice addition (*Fratres*);

Based on the formula T-/+2 for the T-voices addition with T-/+1 incrustations (*Silentium, Arbos*).

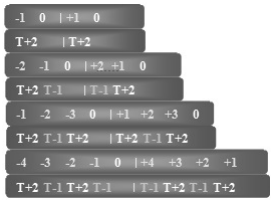
The third group of pieces according to the formula correlation criterion can be ranged regarding the difficulty of the algorithm used. The simplest algorithm is a literal alternation used for both parts of *Tabula Rasa*. The mirrored

order of T+1/T-1 succession in the question-answer related phrases within semi-stanzas from MS3 of *Ludus* or the change of step in formula for T-voice addition (T-/+1 / T-/+2) in the highest T-voice of *Silentium* don't change considerably the settled alternation principle.

A more difficult algorithm is used in the higher T-voice related with middle M-voice from *Arbos*. The formula for T-voice addition depends on the specified numerical value from the range of M-voice. Mainly it consists of the T-1 addition formula, but until and including the *-5* number from M-voice range, all the odd numbers (*-1, -3, -5*) are followed by *tintinnabuli*-tone guided by T-2 addition formula. For the rest of range (until *-9*) the last addition formula is related with all even numbers (*-6, -8*2*).

The most sophisticated algorithms of T-voice addition are used in *Spiegel im Spiegel* in two lower T-voices, thus they must be visualized together with M-voice ranges for better understanding:

Etc.
The lowest T-voice



Etc.
The middle T-voice

The formalization of the lowest T-voice addition algorithm in *Spiegel im Spiegel* can be made with the use of logical formula for M-voice ranges transformations within one stanza, expressed as follows:

1st phrase $-O_s \Rightarrow -O_g\ z \mid +O_s \Rightarrow +O_g\ z$
2nd phrase $-E_g \Rightarrow -O_s\ z \mid +E_g \Rightarrow +O_s\ z$

The T-voice with T-1 formula of addition is added to every *0* note (z) within each stanza, except the third *0* note (z) with T+1 formula of T-voice addition, which appears after range from a greater even number with the minus sign ($-E_g$) to the smallest odd number with the minus sign ($-O_s$).

The middle T-voice algorithm consists of alternation and symmetry principles combination: phrases always begin with T-tone additions according to the formula T+2, which is farther alternated with T-1 formula of T-voice addition. Symmetry with alternation order change occurs on the smallest odd numbers with the plus sign ($+O_s$) and greater even numbers with the plus sign ($+E_g$) beginning the transformation period in each second semi-phrase.

The algorithm for T-voice addition in the highest T-voice is the simplest compared to two lower T-voices and consists only of the T-1 formula for T-voice addition. This T-voice appears as dispersed in accompaniment figure and can be also considered in the context of horizontal correlations too.

Horizontal correlations

Horizontal correlations of T-voice with M-voice are more difficult to perceive at first sight, because they are integrated into the structure which can be treated as melody, but knowing the range of M-voice, T-voice within its structure can be found. There are four cases of such correlations:

1) Simple alternation of M-voice with T-voice (M-voice of violin solos from MS3 of *Ludus*);

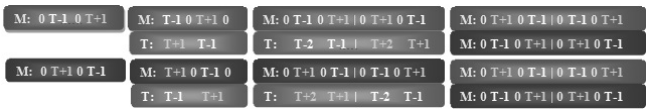
2) Alternation of M-voice with T-voice on specified algorithm (preamble to M-voice of violin solos from MS3 of *Ludus*);

3) T-voice alternation with two dispersed in accompaniment M-voices (*Spiegel im Spiegel*);

4) M-voice germination into T-voice (T-voice of violin solos from MS3 of *Ludus*);

The dynamic changes in the third measure structure from *Ludus* expressed earlier in rhythmic progressions (two violin solos) within each stanza find its application on the level of T and M-voices horizontal correlations. In this case the dynamic changes affect the whole MS3 form development (but within the same two violin solos) and are expressed in combinatory order change of T and M-voices alternation.

The combinatory effect is augmented when formally M-voice (alternated M-voice with T-voice) is followed by separate T-voice. The visual representation of four stanzas looks as follows:



MS3 from *Ludus*

In this visualization each column signifies a new stanza. Horizontal space inside the column stands for semi-stanzas separation. Shades of grey colour express different M-voices. The difference consists of one of two types of preamble use before the main range of numbers in M-voice. The first type of preamble is with the use of sixteens and eighths rhythmic values and the second type is with the use of eighths. The difference between two types of preambles lies in formula construction too, which is described farther in the article.

Vertically added T-voice appears only in the second and third stanzas and in the last stanza it is replaced with M-voice. The formula for T-voice addition undergoes change of the step from -/+1 to -/+2, which appears in the third stanza. The relation of the T-voices added to different semi-stanzas within each stanza has a reversed character.

The relations of horizontally added T-voices within the structure of M-voices belonging to different semi-stanza within each stanza also have a reversed character. Thus, it will be sufficient to understand the logic of T-voice belonging to the first semi-stanzas for understanding the logic of T-voice within the second semi-stanzas. T-voice from the first semi-stanza of the first stanza follows the logic of T-1/T+1 succession and it is repeated in the second and beginning of the third stanzas, but in the last fourth stanza the succession that begins the semi-stanza is reversed. The mirroring principle is inherent in the last two stanzas and resembles the same principle used for vertically added T-voice to the middle M-voice in *Spiegel im Spiegel*. If we represent the primer succession T-1/T+1 as *A* and its reversed version as *B*, we receive the following formalization for the horizontally added T-voice within each first semi-stanza:

A A AUB BUA

Naturally the logical formalization for the horizontally added T-voice within each second semi-stanza will be the reversed version of the first formula:

B B BUA AUB

Preambles formation

The algorithm of two types of preambles formation is more sophisticated than the simple alternation of M and T voices. Also it is developed from the end. That means that the general formula was created initially and placed at the very end of the whole MS3 form. Then its elements where gradually reduced to the beginning of the form. The initial formula can be visually presented as follows:

Full algorithm of preambles from MS3 of *Ludus*

Two ranges from 7 to 0 reflect the structure of two types of preambles. The range with the plus sign is inherent in the first type of preambles and the range with the minus sign – in the second type. The range inherent in the second type is guided by simple alternation principle, but the first type has two reduced elements (T+1) between the pair of even-odd numbers and one added element (also T+1) between the last pair of odd-even numbers.

Percussion

Percussion usually plays the role of a separator between stanzas in forms of Pärt's compositions. It is the case of *Arbos* and *Fratres*. In *Arbos* three bells with a single sound with proper value (rhythmic values *1, 2, 3.5*) for each of M-voices (with rhythmic values for each layer *1, 2, 4*) mark the end of stanza. In *Fratres* the percussion's "separator" has an expression of two measures with repeated rhythmic formula. The organ version of *Mein Weg* has quasi percussion "dots" – eights for the main highest M-voice and quarters for the middle M-voice (Shvets, 2013), separated by pauses of the same rhythmic value from two sides.

Cantus in Memory of Benjamin Britten makes an exception to this rule where the bell voice is not related to the M-voice structure and follows its proper mathematical logic: 3 sounded measures (alternated with 2 measures of pauses) and 3 measures of pauses. After being repeated 11 times the bell voice stops. Then after 22 measures of pauses one last sound appears, becoming the end "dot" of the whole piece. The behaviour of the bell voice could be expressed by the following formula:

(s_m + p_m) * 3) + 2 p_m) * 11) + 22 p_m) + 1s

where *s_m* corresponds to the measure with sound, *p_m* to the measure filled with pauses and *1s* to the one sound.

The relation between number *11* (the times of bell metric structure repetition) and number *22* (number of measures of pauses after those repetitions), is evident, because the number *22* is the number *11* multiplied by *2*.

Conclusion

The presented analysis of the form development in Pärt's compositions has shown that his music uses a wide range of mathematical operations – from linear algebra (algebraic operations and combinatorics) to the elements of mathematical analysis (mathematical unity of sets). Such high level of algorithmization of creative process allows positioning the compositions of Pärt in a context of generative art.

The new methodology of musical processes representation with the use of computer languages semantics was developed. The *if* statement, *for loop* and *while loop* statements were applied to the form structure representation in both senses – as expressions for M-voice development and as representation of regularities for the measure grid construction; *nested for loop* statement was applied for rhythmic values addition regularities. The application of new methodology allowed us to represent the algorithms inherent in the development of Pärt's compositions in an appropriate way, considering the used creative strategies. The compactness of representation according to this methodology corresponds to the conditions of information society and allows farther processing of received algorithm in big data context.

Literature

Ahn, Yun-Kang. *L'analyse musicale computationnelle : rapport avec la composition, la segmentation et la représentation à l'aide des graphes,* PhD thesis, Université Paris VI – Pierre et Marie Curie, Decembre 2009.

Andreatta, A. *Méthodes algébriques en musique et musicologie du XX^e siècle: aspects théoriques, analytiques et compositionnels,* PhD thesis, École des Hautes Études en Sciences Sociales, Decembre 2003.

Brauneiss, L. Musical archetypes: the basic elements of the tintinnabuli style. In: *The Cambridge companion to Arvo Pärt,* Cambridge University Press, 2012, pp. 49–75.

De Paiva Santana, Ch.; Bresson, Jean. Vers la Modélisation des Pensées Musicales: Le cas du "Tintinabuli" d'Arvo Pärt. *Conférence sur la Modélisation Mathématique et Informatique des Systèmes Complexes (COMMISCO 2012)*, Paris, France, 23–25 Octobre 2012, http://repmus.ircam.fr/_media/depaiva/commisco2012.pdf.

Greenbaum, S. *Arvo Pärt's "Te Deum": A compositional watershed*, PhD thesis, The University of Melbourne, June 1999.

Lyotard, J.-F. *La condition postmoderne: le rapport sur savoir*, Editions de Minuit, 1979.

Pärt, A. The truth is very simple. In: *Sovietskaja muzyka* (Russian translation by E. Michaltchenkova, in Russian), 1990, No. 1, pp. 130–132.

Robinson, T. Analyzing Pärt. In: *The Cambridge companion to Arvo Pärt,* Cambridge University Press, 2012, pp. 76–110.

Shenton, A. *The Cambridge companion to Arvo Pärt,* Cambridge University Press, 2012.

Shvets, A. "Mein Weg" of Arvo Pärt in the context of postmodernism. In: *Scientific almanac of the Lviv University. Series: Art Studies* (in Ukrainian), 2012, Issue 11, pp. 31–44.

Shvets, A. Interactive application for visualization of the form of written postmodern music. In: *Proceeding of EVA London 2013. British Computer Society (BCS)*, London, 29th–31st July 2013, p. 302–309.

Zamornikova, K.; Katunyan M. Arvo Pärt's "Fratres": The Prayer in Music. In: *Lietuvos muzikologija*, Vol. 12, 2011, pp. 111–136.

Zhdanovich, K. "Tabula Rasa" of Arvo Pärt as significant degree of creativity and example of tintinnabuli – style. In: *Scientific almanac of the National Musical Academy of P. Tchaikovsky* (in Ukrainian), 2008, Vol. 76, pp. 39–48.

Santrauka

Iki šiol Arvo Pärto kūrybos tyrimuose buvo naudojami skirtingi metodai, nagrinėjantys glaudų grafinių idėjų / eskizų ir garso įgyvendinimo muzikos kūriniuose ryšį (Shenton, 2012). Tokie metodai kaip stiliaus analizė, muzikinė hermeneutika, Schenkerio analizė, aibių teorija, triadinė transformacija ir kiti (Robinson, 2012) gali būti prasmingi tik iš dalies – jie nesuteikia galimybės visiškai suprasti Pärto kūrybinį procesą. Pavyzdžiui, Leopoldo Brauneisso (Brauneiss, 2012) pasiūlyta muzikos archetipų idėja nepasiteisina, kai tą pačią ritminę struktūrą galima rasti ne tik kompozicijoje „Arbos", kuriai, kaip teigia muzikologas, struktūra buvo sukurta, bet ir kituose Pärto kūriniuose – „Mein Weg", „Cantus Benjamino Britteno atminimui", ritminės struktūros veidrodinę versiją – „Silentium" iš „Tabula Rasa". Taigi tradiciniai muzikos metodai nėra visai tinkami analizuojant Pärto kūrinius. Vietoj jų gali būti naudojami skaitmeninės / kompiuterinės muzikos analizės (angl. *computational musicology*) metodai. Šis su technikos atsiradimu siejamas analizės būdas atsirado XX a. ir atlieka muzikos skaitmeninio aspekto ir sąveikų tarp pavaizdavimo lygių, kurie naudoja šį skaitmeninį aspektą, teorinių modelių kūrimo paiešką (Ahn, 2009). Vienas iš skaitmeninės muzikologijos metodų yra algebrinė muzikos analizės kryptis, siūlanti matematinius metodus (Andreatta, 2003).

Nors Pärto estetinė pozicija ir teigia, kad tai, kas gali būti išreikšta matematiškai, neturi nieko bendra su muzika (Pärt, 1990), savo kūriniuose kompozitorius pasitelkia matematiką, visokeriopą visų muzikinių parametrų skaičiavimą, taikomą bendrai struktūrai, melodijos modelių kūrimui, polifoniniam santykiui tarp balsų. Remiantis keliais Pärto muzikos pavyzdžiais, straipsnyje parodomi matematiniai dėsningumai, ypač susiję su formos konstrukcija. Taip pat šiuo straipsniu keliamas tikslas sukurti naują metodą, leidžiantį pavaizduoti muzikinį procesą kaip logines formuluotes panaudojant tokias modernias kompiuterines kalbas kaip „Java", C++, „Python" ar „Processing".

Analizei atrinkti šeši instrumentiniai kūriniai – „Cantus Benjamino Britteno atminimui" (1977), „Arbos" (1977), „Tabula Rasa" (1977), „Fratres" (1977), „Spiegel im Spiegel" (1978) ir „Mein Weg" (1989). Kūrinių pasirinkimą lėmė tai, kad muzikinės jų kalbos nevaržo literatūrinis tekstas ir žmogaus balsas, kurie gali sukelti tam tikrų analizės apribojimų. Pasirinktus kūrinius lengva palyginti jų konstrukcijos / struktūros aspektu, o tai labai svarbu komparatyvinei analizei.

Panaudojant kompiuterinės kalbos semantiką sukurta nauja muzikos procesų pavaizdavimo metodologija. Tokios formuluotės kaip *if*, *for loop* ir *while loop* buvo taikomos vaizduojant formos struktūrą dviem prasmėmis – kaip M-balso plėtojimo išraiška ir kaip bendros konstrukcijos tinklelio dėsningumų pavaizdavimas; formuluotė *nested for loop* panaudota analizuojant adityvinio ritmo verčių dėsningumus. Naujos metodologijos pritaikymas leido pavaizduoti algoritmus, glūdinčius Pärto kūrinių plėtotėje ir susijusius su panaudota komponavimo technika / strategija. Šiam analizės metodui būdingas vaizdavimo glaustumas atitinka informacinės visuomenės sąlygas ir suteikia galimybę gautą algoritmą vėliau apdoroti plačiame duomenų kontekste.

Pärto muzikoje konstruojamos formos analizė parodė, kad šio autoriaus kūriniuose panaudotas platus matematinių operacijų spektras nuo linijinės algebros (algebrinių operacijų ir kombinatorikos) iki matematinės analizės elementų (matematinio aibių vieningumo). Tiriant nustatyta, kad Pärto kūrybiniam procesui būdinga algoritmizacija šio kompozitoriaus kūrinius leidžia priskirti generatyvinio meno sričiai.